Two dimensional phase unwrapping of Interferometric SAR data by means of wavelet technique

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***Abstract*- One of the reliable methods in least-squares method is multigrid technique which overcomes the problem of slow convergence and less-accurate of Gauss-Seidel by transforming problem to coarser grid. It makes a pyramid of grids. Each grid has half the resolution of its predecessor. It uses two restriction and prolongation operators called fine-to-coarse and coarse-to-fine operators respectively. In this research, discrete wavelet decomposition and its reconstruction have been applied on the two operators. One of the assumptions made on this operator is that as long as the wavelet transformation decomposes the 2-D signal to one low frequency and three high frequency components, it should converge faster and more accurate than the multigrid method. This is due to the fact that the transformation of only low frequency component would suffice rather than transforming the whole grid to coarser grid. The idea has been implemented and tested on simulation data and the results confirm the assumption. In this paper the results of implementation of various wavelet filters and also multigrid techniques on various simulation data (with and without noise) are presented. In all cases, wavelet techniques have shown improved results than multigrid techniques.**

I. INTRODUCTION

Interferometric Synthetic Aperture Radar (InSAR) is a technique that uses two or more SAR images over the same area for extracting high-resolution digital terrain data. The technique relies on the measurement of the phase of the echoed signal rather than its amplitude, as found in conventional imaging radar system. The extreme sensitivity of the technique to altitude changes, high spatial resolution and broad swath coverage makes it an extensive and accurate measurement means in many fields; namely earthquake monitoring, erosion studies, mining prospecting, and military tactics. The technique brings strong advantages such as independency of natural illumination or recognizable targets over classical stereoscopic optical imaging.

Nevertheless, InSAR presents very difficult stages like phase unwrapping. Phase unwrapping is the key problem in building the elevation map of a scene from interferometric synthetic aperture radar system data. Due to the nature of SAR imaging, they do not contain information about the absolute phase of the returning radar echoes, but the phase is wrapped to the interval $\left[-\pi, \pi\right]$. Reconstruction of the absolute phase from the wrapped phase value is called phase unwrapping.

A variety of approaches to 2-D phase unwrapping have been proposed recently. They can be classified to local and global or path-following and least-squares methods respectively. The first method is based on the identification of residues, local errors in the measured phase caused by signal noise or by actual discontinuities, and the definition of suitable branch cuts to prevent any integration path crossing these cuts. The estimated neighboring pixel differences of unwrapped phase are integrated along paths avoiding the branch cuts where these estimated differences are inconsistent [1]. The problems of this approach are the definition of suitable branch cuts and the time consuming computations. The least-squares methods that are global methods are introduced in next section. After considering wavelet method in section II we give some example on it and compare it with multigrid method.

II. ALGORITHM

Let us assume that we know the phase of interferogram, $\psi_{i,j}$, that is known only between $-\pi$ and π. We want to determine the unwrapped phase value, $\phi_{i,j}$, at the same grid locations:

$$\psi_{i,j} = \phi_{i,j} + 2\pi.k \quad \text{k is an integer} \quad -\pi < \psi_{i,j} < \pi \qquad (1)$$

The least square approach for phase unwrapping obtained this unwrapped phase value by minimizing the difference between the discrete partial derivatives of the wrapped phase data and the discrete partial derivatives of the unwrapped solution [2].
We define the following partial derivatives of the wrapped phase data as [2]:

$$\Delta x_{i,j} = W\{\psi_{i,j+1} - \psi_{i,j}\}, \ \Delta y_{i,j} = W\{\psi_{i,j+1} - \psi_{i,j}\} \quad (2)$$

The difference between these partial derivatives (2) and the partial derivatives of solution must be minimized in least square sense:

$$\sum_i \sum_j (\phi_{i,j} - \phi_{i-1,j} - \Delta x_{i.j})^2 + \sum_i \sum_j (\phi_{i,j} - \phi_{i,j-1} - \Delta y_{i.j})^2 \quad (3)$$

Differentiating the above sum with respect to $\phi_{i,j}$ and setting the result equal to zero, the following equation is obtained:

$$(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}) + (\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}) = \rho_{i,j} \quad (4)$$

Where $\rho_{i,j}$ is equal to:

$$\rho_{i,j} = (\Delta x_{i+1,j} - \Delta x_{i,j} + \Delta y_{i+1,j} - \Delta y_{i,j}) \qquad (5)$$

Equation (4) is a discretization of the Poisson's equation (6) in a rectangular grid [2]:

$$\frac{\partial^2}{\partial x^2}\phi(x,y) + \frac{\partial^2}{\partial y^2}\phi(x,y) = \rho(x,y) \qquad (6)$$

Writing above equation in matrix format yields the following equation:

$$A\phi = \rho \qquad (7)$$

Which A is a sparse matrix and ϕ is the solution of phase unwrapping. The classical method for solving the Poisson's equation is called Gauss-Seidel relaxation. Due to its extremely slow convergence, Gauss-Seidel relaxation is not a practical method but it is the base of multigrid [3] and wavelet method.
Gauss-Seidel relaxation is essentially a local smoothing operator that removes the high-frequency components of the error very quickly but the low-frequency components extremely slowly [3]. Multigrid techniques overcome this limitation by transforming low-frequency components of error into high-frequency components which can be removed quickly by Gauss-Seidel relaxation. This is accomplished by transforming the problem to coarser grid. It make a pyramid of grids, each grid has half the resolution of its predecessor and uses two restriction and prolongation operators called fine-to-coarse and coarse-to-fine operator, respectively[3]:

$$c_{i,j} = \frac{1}{16}(f_{2i-1,2j-1} + f_{2i+1,2j-1} + f_{2i-1,2j+1} + f_{2i+1,2j+1}) + \frac{1}{8}(f_{2i,2j-1} + f_{2i,2j+1} + f_{2i-1,2j} + f_{2i+1,2j}) + \frac{1}{4} f_{2i,2j} \quad (8)$$

$$\begin{aligned} f_{2i,2j} &= c_{i,j} \\ f_{2i+1,2j} &= \frac{1}{2}(c_{i,j} + c_{i+1,j}) \\ f_{2i,2j+1} &= \frac{1}{2}(c_{i,j} + c_{i,j+1}) \\ f_{2i+1,2j+1} &= \frac{1}{4}(c_{i,j} + c_{i+1,j} + c_{i,j+1} + c_{i+1,j+1}) \end{aligned} \qquad (9)$$

Where $f_{i,j}$ is the "fine" grid and $c_{i,j}$ is the "coarse" grid.

In this research the pair of wavelet transform, decomposition (analysis) and reconstruction (synthesis), have been applied on the two operators. For the decomposition in stage one, we first convolve the rows of the image S with low-pass and high-pass filters (Lo_D and Hi_D) and discard the odd-numbered columns (downsample) of the two resulting arrays. The columns of each of the N/2-by-N arrays are then convolved with low-pass and high-pass filters and the odd numbers rows are discarded. The result is the four N/2-by-N/2 arrays required for the stage of transform [4]. In Fig.1 cA_0 is equal to S for the decomposition initialization.

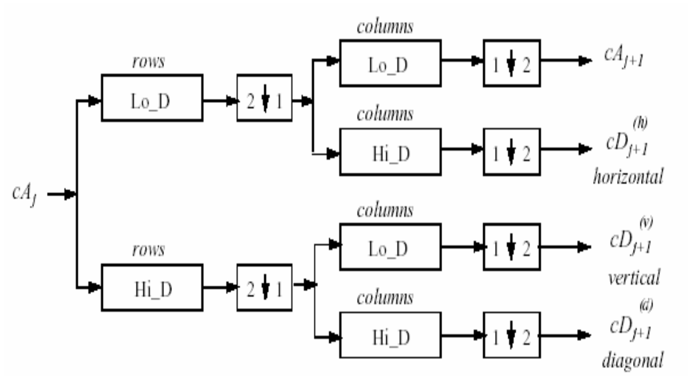

Figure 1.Decomposition step of discrete wavelet transform

Just like the multigrid method [3] after relaxing v_1 time on the equation $A\phi_N = \rho_N$ with initial guess ϕ_N we transfer the residual error of the above equation with decomposition step of Discrete Wavelet Transform (DWT) to be solved to the coarser grid. Our coarser grid composed of one low-frequency and three high-frequency components. In this stage relaxation is done on the residual equation $Ae = r$ in the low-frequency component grid, with the initial guess $e = 0$ where r is the known residual error, $r = \rho - A\hat{\phi}$. The resulting solution on the lower component of coarser grid can be regarded in turn as an intermediate solution whose residual error is transformed to the next coarser grid with DWT. This process continues to $J < \log_2^N$ to the coarsest grid, then the solution transfer to finer grid by reconstruction analysis of DWT (for more detail on wavelet transform see Ref.4) and added to the approximation $\hat{\phi}$ to yield a better solution on the finer grid. Like multigrid algorithm we can move along a V-cycle or W-cycle [3]. This algorithm can be extended to weighted case that again uses decomposition and reconstruction step of DWT instead of restriction and prolongation operators of multigrid.

III. EXPERIMENTAL RESULTS

In order to test the performance of the illustrated algorithms we consider the simulated phase pattern of 256×256 pixels that is shown in figure 2. We wrapped this function to generate an interferogram (Fig. 3). The reconstructed phase pattern via Gauss-Seidel, multigrid and wavelet method with the same number of iterations (100 iterations) are shown in Figs. 4-6. Standard deviation error for multigrid and wavelet methods is 2.88 and 2.10, respectively. Biorthogonal wavelet with effective length of 17 and 3 is used for low-frequency and high-frequency filters. Standard deviation error of other filters such as Discrete Meyer and Daubechies (order 15) is 2.22 and 2.15 respectively. Finding the best filter for a specific region remains as further research.

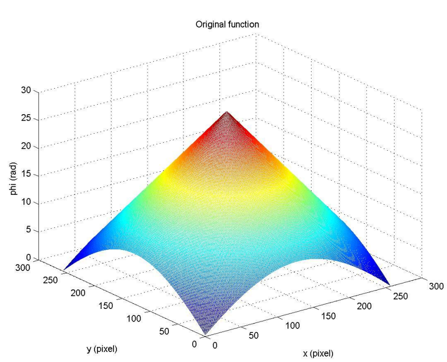

Figure 2. Simulated phase pattern

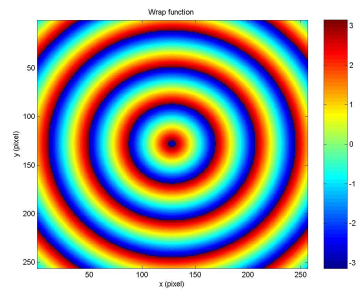

Figure 3. Interferogram

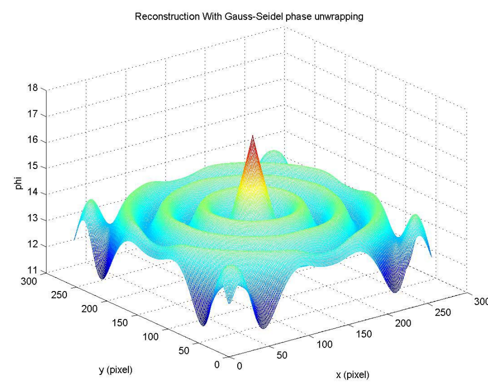

Figure 4. Reconstruction with Gauss-Seidel

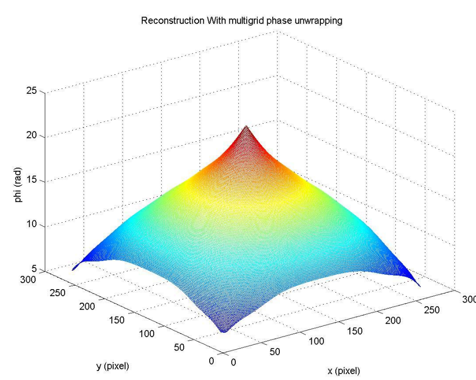

Figure 5. Reconstruction with multigrid

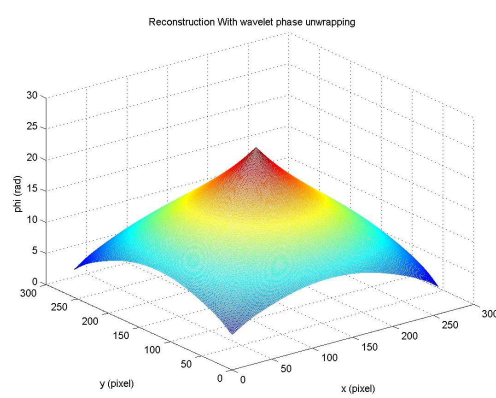

Figure 6. Reconstruction with wavelet

In other experiment a terrain model is generated with fractal (Fig. 7) and its corresponding interferogram contain a pattern of fringes is shown in Fig.8. In this case a weighted wavelet and multigrid method is used. Figs. 9-10 show the weights for row and column derivatives by the phase derivatives variance algorithm. Fig. 11 shows the reconstruction phase pattern of wavelet method. In this case the RMS error and standard deviation for wavelets are 0.62 and 0.59 while these values for multigrid are 0.66 and 0.64 that shows a little better solution of wavelet in weighted case. As Pritt explained in Ref.[5] least squares solutions are not congruent to the wrapped input phase, so we did a post processing step on two results and RMS errors after this step are 0.13 and 0.24 for wavelet and multigrid, respectively.

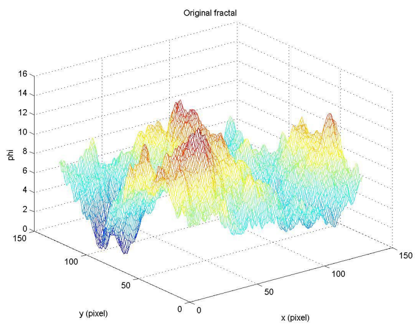

Figure 7. Simulated phase with fractal

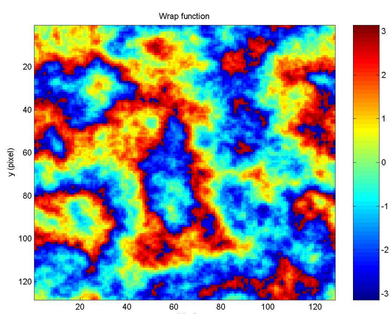

Figure 8. Interferogram of fractal

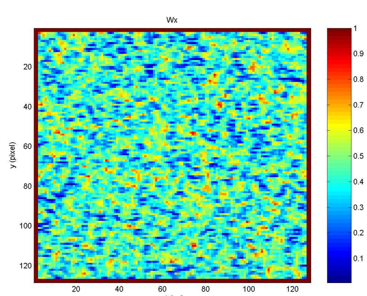

Figure 9. Row weights

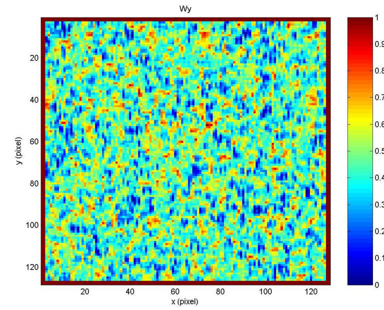

Figure 10. Column weights

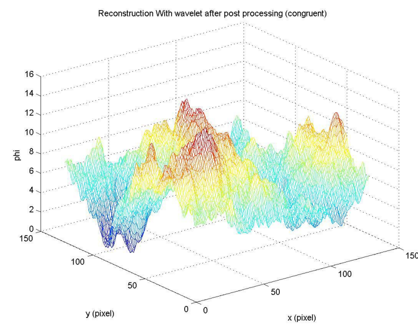

Figure 11. Reconstruction with wavelet

IV. CONCLSION

An approach for phase unwrapping has been described based on wavelet transformation. Experiments show the better accuracy of wavelet method in comparison with multigrid. In addition wavelet has the potential of using various filters that must be tested in further research.

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